

Equations

Walking to help the 'Kids' hotline'

<i>Sponsor's name</i>	<i>Address</i>	<i>City</i>	<i>Amount per km</i>
1 James McLennan	12 Thomas St	Hampton	\$2.00
2 Gillian McLennan	12 Thomas St	Hampton	\$2.00
3 Tania Smith	18 Thomas St	Hampton	\$3.00
4 Tony Intihar	51 Rochester St	Richmond	\$1.50
5 Nadia Intihar	51 Rochester St	Richmond	\$1.00
6 Emma Wu	7 Smith Ave	Northcote	\$2.50
7 Trent Taylor	25 Perth St	Eltham	\$3.00

6

To raise money for the 'Kids' hotline' your school has organised a walk-a-thon. A copy of the sponsor form for your friend, Alexandra is shown on the left. If she raised \$75.00, how far did she walk? By the end of this chapter you will be able to answer this easily by writing the information as an equation that can be solved.



Using inverse operations

Here is a simple number puzzle:

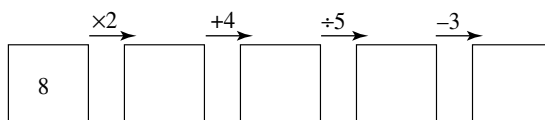
I am thinking of a number — when I add 5 to the number my answer is 18. What number am I thinking of?

We can write this puzzle as an equation (an expression containing the equals sign), using a pronumeral to stand for the unknown number. In this case, using x to stand for the number, we would write: $x + 5 = 18$.

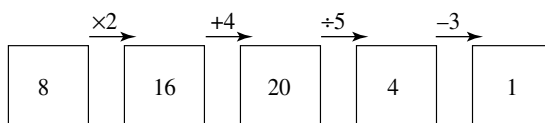
To solve this equation we must find the value of x . You can probably see that x must be 13 so we say that $x = 13$ is the solution of the equation. This is called solving the equation by inspection, and it works for very easy equations. For harder equations we will need to find better methods.

Flow charts

We will use diagrams called *flow charts* to help us to organise harder equations. Here is an example of a flow chart:

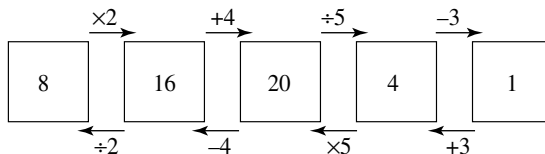


The 8 at the front is called the input number, and by carrying out each operation we can move through the flow chart until we reach the last number, called the output number. If we fill in this flow chart, we see that the output number is 1.



Backtracking

It is possible to travel in either direction through a flow chart. Working backwards, against the arrows, is called *backtracking*. For the flow chart above, we can backtrack by showing the inverse operations.



Each arrow carries an operation, for example -3 or $\div 5$, and when you backtrack you must carry out the opposite or *inverse operation*.

Inverse operations are opposite operations so:

$+3$ is the inverse of -3

$\times 5$ is the inverse of $\div 5$

-4 is the inverse of $+4$

$\div 2$ is the inverse of $\times 2$.

WORKED Example 1

Write an equation to represent each of these puzzles.

I am thinking of a number.

a When I multiply the number by 8 the answer is 24.

b When I divide the number by 5 the answer is 7.

THINK

- a**
- 1 Use a pronumeral to describe the number.
 - 2 Multiply the number by 8.
 - 3 Write the equation.

b

- 1 Use a pronumeral to describe the number.

2 Divide the number by 5.

3 Write the equation.

WRITE

a Let m be the number.

$$m \times 8 = 8m$$

$$8m = 24$$

b Let t be the number.

$$t \div 5 = \frac{t}{5}$$

$$\frac{t}{5} = 7$$

WORKED Example 2

Solve the following equations by inspection.

a $\frac{w}{3} = 4$

b $h - 9 = 10$

THINK

- a**
- 1 Write down the equation.
 - 2 Think of a number which when divided by 3 gives 4. Try 12.
 - 3 So w must be 12.

- b**
- 1 Write down the equation.
 - 2 Think of a number which equals 10 when 9 is subtracted from it. Try 19.
 - 3 So h must be 19.

WRITE

a $\frac{w}{3} = 4$

$$12 \div 3 = 4$$

$$w = 12$$

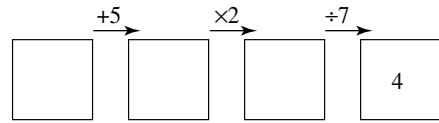
b $h - 9 = 10$

$$19 - 9 = 10$$

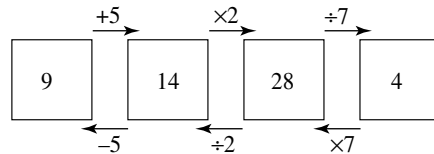
$$h = 19$$

WORKED Example 3

Use backtracking and inverse operations to find the input number in this flow chart.

**THINK**

- 1 Fill in the numbers as you backtrack.
The inverse of $\div 7$ is $\times 7$
($4 \times 7 = 28$).
- 2 The inverse of $\times 2$ is $\div 2$
($28 \div 2 = 14$).
- 3 The inverse of $+ 5$ is $- 5$
($14 - 5 = 9$).

WRITE**remember**

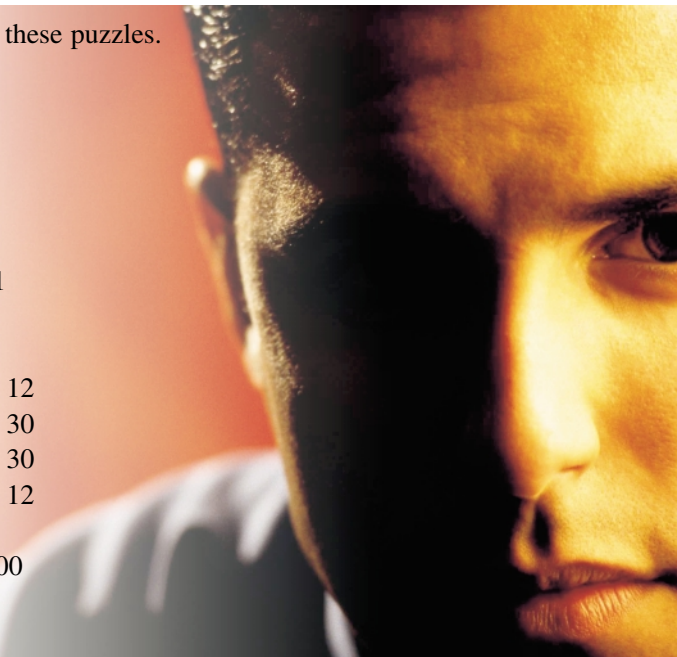
1. Flow charts help us to organise harder equations.
2. We can use *backtracking* and *inverse operations* to solve equations.

EXERCISE 6A**Using inverse operations****WORKED Example 7**

- 1 Write an equation to represent each of these puzzles.

I am thinking of a number:

- a when I add 7 the answer is 11
- b when I add 3 the answer is 5
- c when I add 12 the answer is 12
- d when I add 5 the answer is 56
- e when I subtract 7 the answer is 1
- f when I subtract 11 the answer is 11
- g when I subtract 4 the answer is 7
- h when I subtract 8 the answer is 0
- i when I multiply by 2 the answer is 12
- j when I multiply by 6 the answer is 30
- k when I multiply by 5 the answer is 30
- l when I multiply by 6 the answer is 12
- m when I divide by 7 the answer is 1
- n when I divide by 3 the answer is 100
- o when I divide by 5 the answer is 2
- p when I divide by 7 the answer is 0.





WORKED Example
2

2 Solve the following equations by inspection.

a $x + 7 = 18$

b $y - 8 = 1$

c $3m = 15$

d $\frac{m}{10} = 3$

e $w + 25 = 26$

f $m - 1 = 273$

g $4w = 28$

h $\frac{k}{5} = 0$

i $b + 15 = 22$

j $\frac{w}{3} = 6$

k $5k = 20$

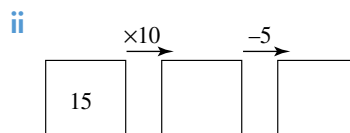
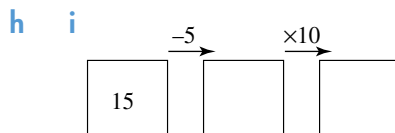
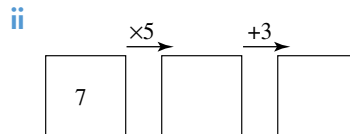
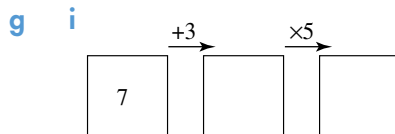
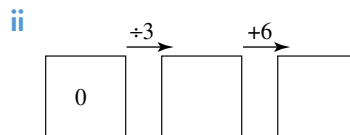
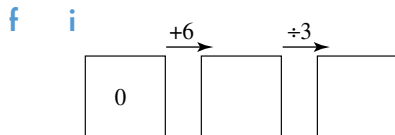
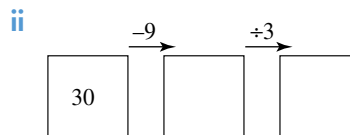
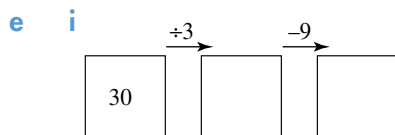
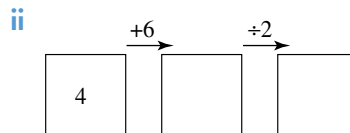
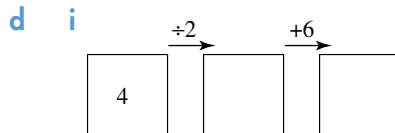
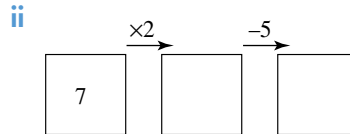
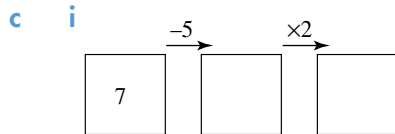
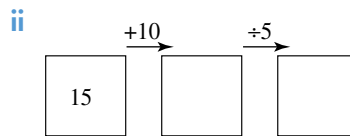
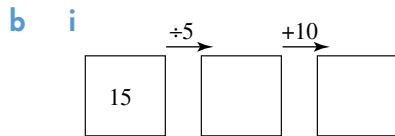
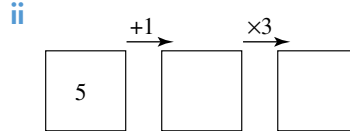
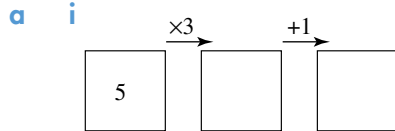
l $h - 14 = 11$

m $b - 2.1 = 6.7$

n $\frac{c}{3} = 1.4$

o $5x = 14$

3 Complete the following flow charts to find the output number.



4 a In question 3, what did you notice about each pair of flow charts?

b Does changing the order of operations affect the end result?

5 a Complete the 2 statements below:

Adding and _____ are inverse operations.

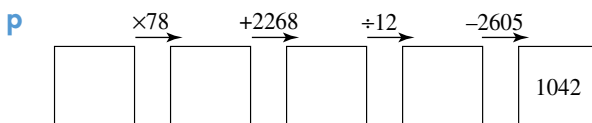
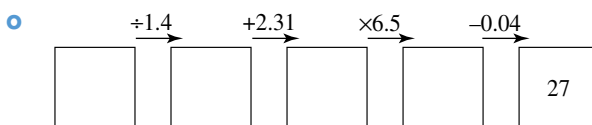
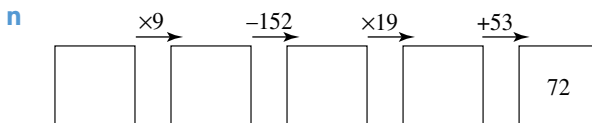
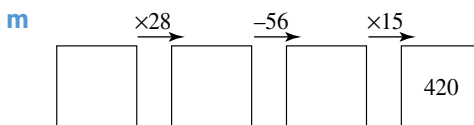
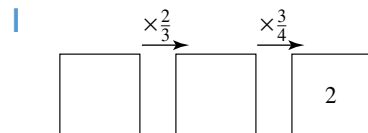
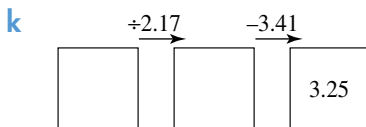
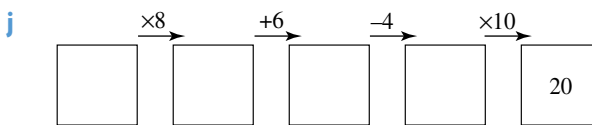
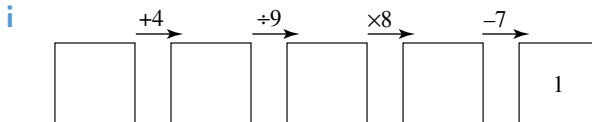
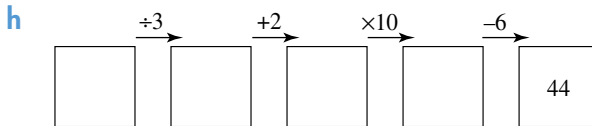
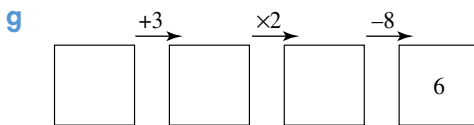
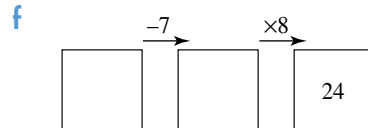
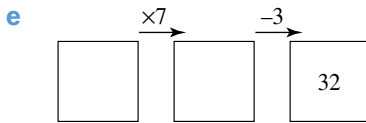
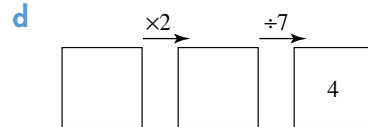
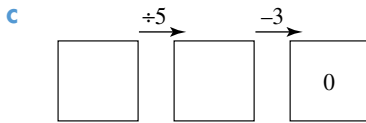
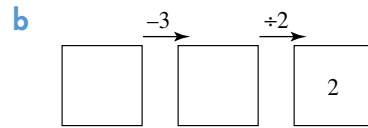
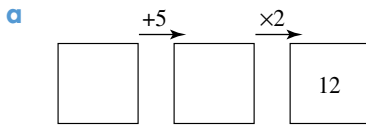
_____ and dividing are inverse operations.

b Can you think of any other pairs of inverse operations?



WORKED
Example
3

6 Use backtracking and inverse operations to find the input number in each of these flow charts.



Joke time



Solve the equations below to find the puzzle answer code.

What tools do we use in arithmetic?



9

14

17

16

19

12

17

19

8

6

10

$A + 7 = 18$

$A = \text{[]}$

$G - 12 = 3$

$G = \text{[]}$

$12 + B = 15$

$B = \text{[]}$

$H - 7 = 0$

$H = \text{[]}$

$27 = C + 9$

$C = \text{[]}$

$16 = I - 3$

$I = \text{[]}$

$5 = 4 + D$

$D = \text{[]}$

$11 = K - 2$

$K = \text{[]}$

$E + 38 = 46$

$E = \text{[]}$

$L - 13 = 4$

$L = \text{[]}$

Why was Christopher Columbus a crook?



7

8

1

5

14

3

17

8

18

6

5

10

10

8

1

16

7

8

11

16

17

11

4

16

19

18

$3M = 27$

$M = \text{[]}$

$S \div 5 = 2$

$S = \text{[]}$

$5N = 20$

$N = \text{[]}$

$\frac{T}{4} = 4$

$T = \text{[]}$

$12O = 60$

$O = \text{[]}$

$\frac{U}{2} = 7$

$U = \text{[]}$

$7P = 84$

$P = \text{[]}$

$W \div 4 = 5$

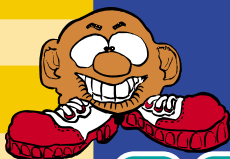
$W = \text{[]}$

$9R = 54$

$R = \text{[]}$

$\frac{Y}{2} = 1$

$Y = \text{[]}$



What walks on its head all day?

11

1

6

11

20

19

4

15

12

19

4

10

16

14

18

13

19

4

2

5

14

6

10

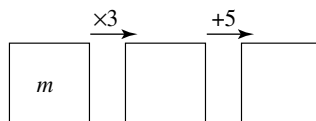
7

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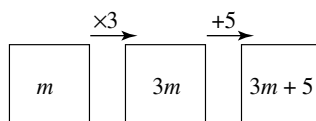
8

Building up expressions

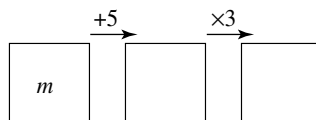
We can use flow charts to construct algebraic expressions. Here is an example, with an input number equal to m .



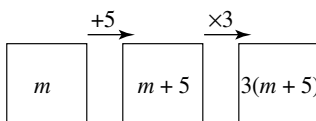
Multiplying m by 3 gives us $3m$, and then adding 5 gives $3m + 5$ which is the output number.



Compare it with this flow chart.



Adding 5 to m gives us $m + 5$, but to show that we are multiplying all of $m + 5$ by 3 we need to place $m + 5$ inside a pair of brackets. Now, $(m + 5) \times 3$ is written as $3(m + 5)$. A pair of brackets is a type of grouping device. It groups $m + 5$ together.

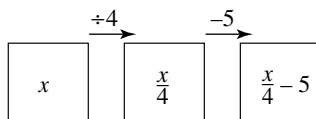


Remember that there is a difference between $2 + 5 \times 3$ and $(2 + 5) \times 3$. So the flow charts which gave us the expressions $3m + 5$ and $3(m + 5)$ contained the same operations, but they were carried out in a different order.

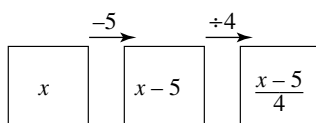
A different grouping device

Compare these two flow charts.

Flow chart A



Flow chart B



Can you see the brackets or grouping device in flow chart B? It is the division line (called a vinculum) drawn beneath $x - 5$ and we must take care that it runs beneath all of $x - 5$.

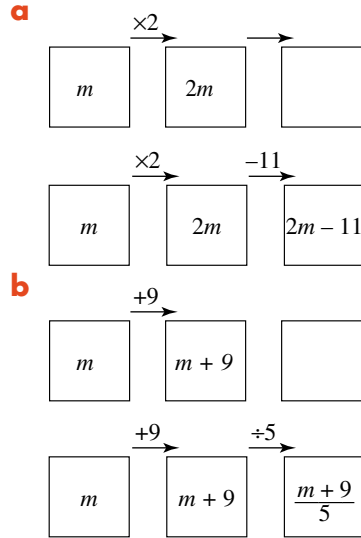
WORKED Example 4

Draw a flow chart whose input number is m and whose output number is given by the expressions: **a** $2m - 11$ **b** $\frac{m+9}{5}$.

THINK

- a**
- 1 The first step is to obtain $2m$.
 - 2 This is followed by subtracting 11.
- b**
- 1 The expression $m + 9$ is grouped as in a pair of brackets, so we must obtain this part first.
 - 2 Then everything is divided by 5.

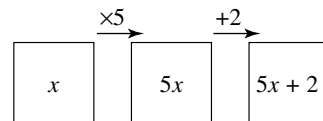
WRITE



As well as constructing an algebraic expression using a flow chart, we can backtrack to our input number using inverse operations.

WORKED Example 5

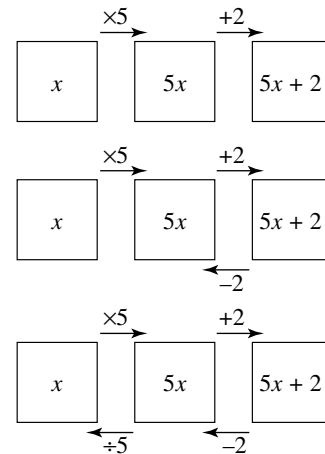
Complete the following flow chart by writing in the operations which must be carried out in order to backtrack to x .



THINK

- 1 Copy the flow chart and look at the operations that have been performed.
- 2 The inverse operation of $+ 2$ is $- 2$. Show this on the flow chart.
- 3 The inverse operation of $\times 5$ is $\div 5$. Show this on the flow chart.

WRITE



remember

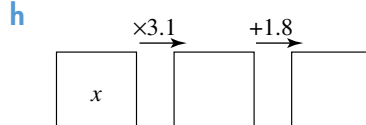
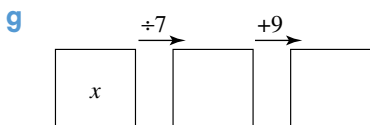
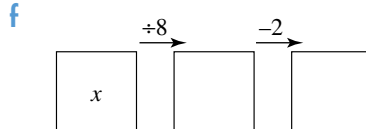
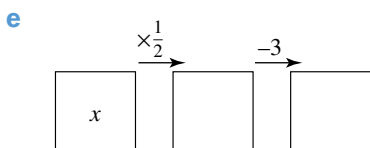
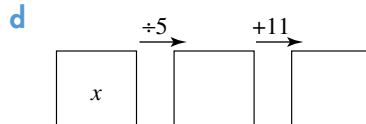
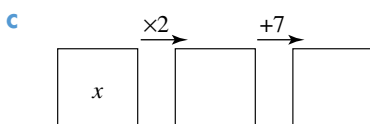
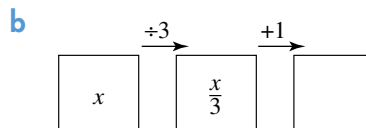
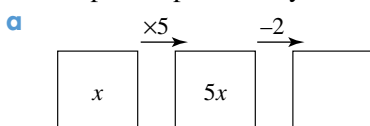
1. We can use flow charts to construct algebraic expressions.
2. As well as constructing an algebraic expression using a flow chart, we can backtrack to our input number using inverse operations.

EXERCISE

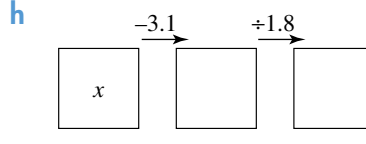
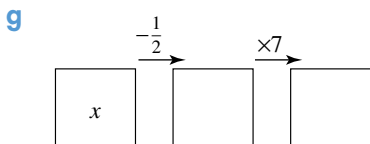
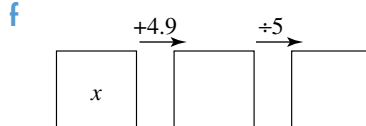
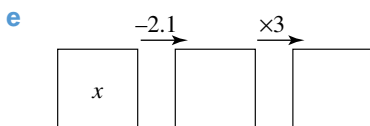
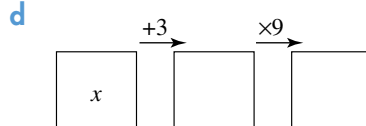
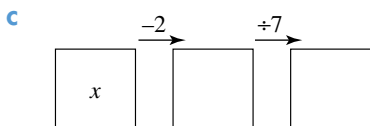
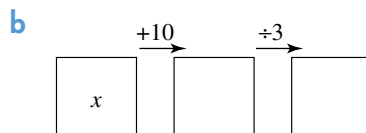
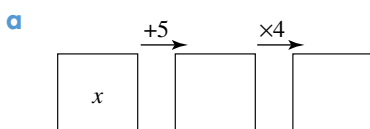
6B

Building up expressions

- 1 Build up an expression by following the instructions on the flow charts.



- 2 Build up an expression by following the instructions on the flow charts. You will need to use a grouping device, such as a set of brackets or a vinculum. For example $2(x + 3)$ or $\frac{x-5}{4}$.



3 Copy and complete the following flow charts by filling in the missing expressions.

<p>a</p> $\boxed{x} \xrightarrow{+2} \boxed{} \xrightarrow{\times 6} \boxed{}$	<p>b</p> $\boxed{x} \xrightarrow{\times 3} \boxed{} \xrightarrow{-2} \boxed{}$
<p>c</p> $\boxed{x} \xrightarrow{-8} \boxed{} \xrightarrow{\div 12} \boxed{}$	<p>d</p> $\boxed{x} \xrightarrow{\times 4} \boxed{} \xrightarrow{+1} \boxed{}$
<p>e</p> $\boxed{x} \xrightarrow{\div 7} \boxed{} \xrightarrow{+3} \boxed{}$	<p>f</p> $\boxed{x} \xrightarrow{-3} \boxed{} \xrightarrow{\div 5} \boxed{}$
<p>g</p> $\boxed{x} \xrightarrow{+5} \boxed{} \xrightarrow{\times 6} \boxed{} \xrightarrow{-2} \boxed{}$	<p>h</p> $\boxed{x} \xrightarrow{\times 3} \boxed{} \xrightarrow{+4} \boxed{} \xrightarrow{\div 7} \boxed{}$
<p>i</p> $\boxed{x} \xrightarrow{-8} \boxed{} \xrightarrow{\div 5} \boxed{} \xrightarrow{+9} \boxed{}$	<p>j</p> $\boxed{x} \xrightarrow{\div 3} \boxed{} \xrightarrow{+4} \boxed{} \xrightarrow{\times 6} \boxed{}$
<p>k</p> $\boxed{x} \xrightarrow{\times 11} \boxed{} \xrightarrow{-2} \boxed{} \xrightarrow{\times 4} \boxed{}$	<p>l</p> $\boxed{x} \xrightarrow{\times 3} \boxed{} \xrightarrow{\div 3} \boxed{} \xrightarrow{+2} \boxed{}$

WORKED Example 4

4 Draw a flow chart whose input number is x and whose output is given by the following expressions.

- | | | |
|-----------------------------|----------------------------|--|
| a $5x + 9$ | b $2(x + 1)$ | c $\frac{x}{6} + 4$ |
| d $\frac{x - 8}{7}$ | e $12(x - 7)$ | f $\frac{x}{5} - 2$ |
| g $7x - 12$ | h $\frac{x + 6}{3}$ | i $3(x + 7) - 5$ |
| j $\frac{3x + 7}{2}$ | k $4(3x + 1)$ | l $3\left(\frac{x}{5} + 6\right)$ |

WORKED Example 5

5 Complete the following flow charts by writing in the operations which must be carried out in order to backtrack to x .

<p>a</p> $\boxed{x} \xrightarrow{\times 7} \boxed{7x} \xrightarrow{+3} \boxed{7x + 3}$ <p style="text-align: center;">← ←</p>	<p>b</p> $\boxed{x} \xrightarrow{-2} \boxed{x - 2} \xrightarrow{\times 5} \boxed{5(x - 2)}$ <p style="text-align: center;">← ←</p>
<p>c</p> $\boxed{x} \xrightarrow{\div 2} \boxed{} \xrightarrow{+1} \boxed{\frac{x}{2} + 1}$ <p style="text-align: center;">← ←</p>	<p>d</p> $\boxed{x} \xrightarrow{-5} \boxed{} \xrightarrow{+4} \boxed{\frac{x - 5}{4}}$ <p style="text-align: center;">← ←</p>

e

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{2(x+7)}$$

g

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\frac{x+8}{3}}$$

i

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{8x-3}$$

k

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{3(x+55)}$$

m

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{7x-5}$$

f

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\frac{x}{6}-3}$$

h

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{4(x+8)}$$

j

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\frac{x-7}{5}}$$

l

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\frac{x}{9}+2.1}$$

n

$$\boxed{x} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{\quad} \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \boxed{4(x+8)+5}$$

History of mathematics

ROBERT RECORDE (1510–1558)

During his life . . .

The astronomer Copernicus proposed his idea that the planets revolve around the sun; the Portuguese navigator Vasco da Gama discovered the sea route from Western Europe to the East; Michelangelo painted his famous frescoes on the ceiling of the Sistine Chapel.

Robert Recorde was born in Tenby, Wales and was educated at Oxford and Cambridge. He became the physician to King Edward VI and Queen Mary.

Recorde was principally responsible for establishing the English School of Mathematics and first introduced algebra to England. He is best known for inventing the 'equals' symbol '=' which appears in his book *The Whetstone of Witte* (1557). He thought that the symbol with two parallel line segments was the most appropriate as, in his words, 'noe 2 thynges can be moare equalle'. The symbol '=' was not immediately popular.

The symbol '||' was used by some and 'ae' from the word 'aequalis' meaning equal, was also widely used until the 1700s.

Robert Recorde wrote many other textbooks. Another one was called *The Grounde of Artes* (1540) in which he discusses operations with Arabic numerals, computation with counters, proportion and fractions.

Recorde died in King's Bench prison in Southwark, England where he was sentenced and imprisoned for debt. Although no official record remains of his other crimes, some historians think he was guilty of more serious offences.

Questions

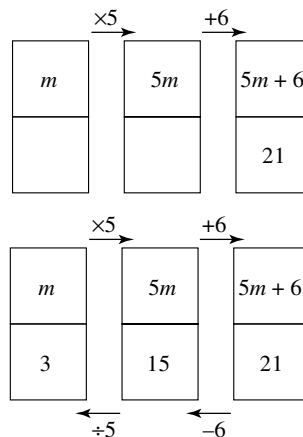
1. What symbol did Robert Recorde invent?
2. Where does 'ae' come from and what does it mean?
3. How old was Robert Recorde when he died?

Solving equations using backtracking

We can now solve harder puzzles and equations by drawing a flow chart and then backtracking. For example, the following problem can be expressed on a flow chart.

I am thinking of a number. If I multiply it by 5 and then add 6 my answer is 21. What is the number? I will call the number m .

By backtracking we can see that m (my number) must be 3.



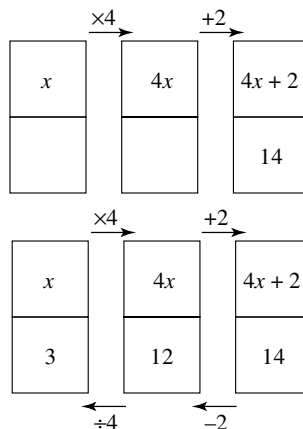
WORKED Example 6

Draw a flow chart to represent the following puzzle and then solve it by backtracking. I am thinking of a number. When I multiply it by 4 and then add 2 the answer is 14.

THINK

- 1 Build an expression using x to represent the number.
Start with x , multiply by 4 and add 2.
The output number is 14.
- 2 Backtrack to find the value of x .
The inverse operation of $+ 2$ is -2
($14 - 2 = 12$).
The inverse operation of $\times 4$ is $\div 4$
($12 \div 4 = 3$).
- 3 State the answer.

WRITE



So $x = 3$. The number is 3.

WORKED Example 7

Solve the following equations by backtracking.

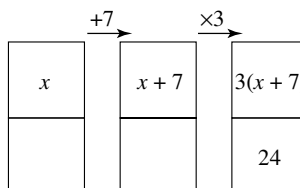
a $3(x + 7) = 24$

b $\frac{x}{3} + 5 = 6$

THINK

- a 1 Build the expression on the left-hand side of the equation.
Start with x , add 7 and then multiply by 3.
The output number is 24.

WRITE



Continued over page

THINK

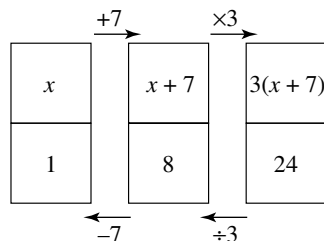
- 2** Backtrack to find x .
 The inverse operation of $\times 3$ is $\div 3$
 ($24 \div 3 = 8$).
 The inverse operation of $+ 7$ is $- 7$
 ($8 - 7 = 1$).

- 3** State the answer.

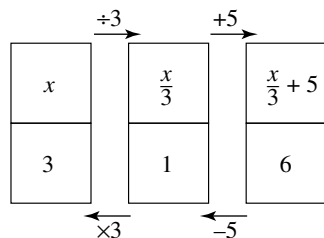
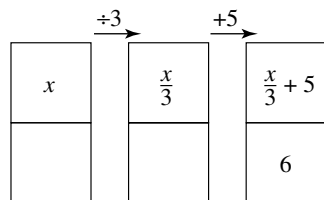
- b 1** Build the expression on the left-hand side of the equation.
 Start with x , divide by 3 and then add 5.
 The output number is 6.

- 2** Backtrack to solve for x .
 The inverse operation of $+ 5$ is $- 5$
 ($6 - 5 = 1$).
 The inverse operation of $\div 3$ is $\times 3$
 ($1 \times 3 = 3$).

- 3** State the answer.

WRITE

$$x = 1$$

b

$$x = 3$$

WORKED Example 8

Simplify and then solve the following equation by backtracking.

$$5x + 13 + 2x - 4 = 23$$

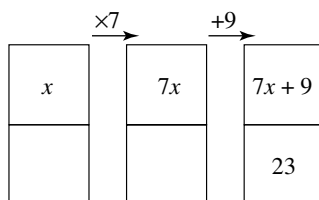
THINK

- 1** Simplify by adding the like terms together on the left-hand side of the equation.
 $5x + 2x = 7x$, $13 - 4 = 9$
- 2** Draw a flow chart and build the expression $7x + 9$.
 Start with x , multiply by 7 and add 9.
 The output number is 23.

WRITE

$$5x + 13 + 2x - 4 = 23$$

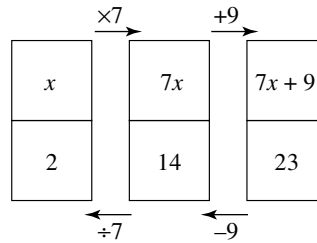
$$7x + 9 = 23$$



THINK

- 3 Backtrack to solve for x .
 The inverse operation of $+ 9$ is $- 9$
 ($23 - 9 = 14$).
 The inverse operation of $\times 7$ is $\div 7$
 ($14 \div 7 = 2$).

- 4 State the answer.

WRITE

$$x = 2$$

remember

We can solve equations by drawing a flow chart and backtracking.

EXERCISE 6C**Solving equations using backtracking**

- 1 Draw a flow chart to represent each of the following puzzles and then solve them by backtracking.

I am thinking of a number.

- a When I multiply it by 2 and then add 7 the answer is 11.
- b When I add 3 to it and then multiply by 5 the answer is 35.
- c When I divide it by 4 and then add 12 the answer is 14.
- d When I add 5 to it and then divide by 3 the answer is 6.
- e When I subtract 7 from it and then multiply by 6 the answer is 18.
- f When I multiply it by 8 and then subtract 11 the answer is 45.
- g When I subtract 4 from it and then divide by 9 the answer is 7.
- h When I divide it by 11 and then subtract 8 the answer is 0.
- i When I add 5 to it and then multiply by 2 the answer is 12.
- j When I multiply it by 6 and then add 4 the answer is 34.
- k When I multiply it by 5 and then subtract 10 the answer is 30.
- l When I subtract 3.1 from it and then multiply by 6 the answer is 13.2.
- m When I divide it by 8 and then subtract 0.26 the answer is 0.99.
- n When I divide it by 3.7 and then add 1.93 the answer is 7.62.
- o When I add $\frac{2}{5}$ to it and then divide by 6 the answer is $\frac{4}{5}$.
- p When I subtract $\frac{3}{4}$ from it and then divide by $\frac{2}{3}$ the answer is $\frac{1}{6}$.



2 Draw a flow chart and use backtracking to find the solution to the following equations.

a $5x + 7 = 22$

b $9y - 8 = 1$

c $3m - 7 = 11$

d $4x + 12 = 32$

e $8w + 2 = 26$

f $11m - 1 = 274$

g $4w + 5.2 = 28$

h $6b - \frac{5}{9} = \frac{2}{3}$

i $2a + \frac{1}{3} = \frac{3}{5}$

3 Solve the following equations by backtracking.

a $3(x + 7) = 24$

b $2(x - 7) = 22$

c $5(x - 15) = 40$

d $11(x + 5) = 99$

e $6(x + 9) = 72$

f $3(x - 11) = 3$

g $4(w + 5.2) = 26$

h $6(b - \frac{5}{9}) = \frac{2}{3}$

i $8(x - \frac{1}{4}) = \frac{4}{5}$

4 Solve the following equations by backtracking.

a $\frac{x}{3} + 5 = 6$

b $\frac{x}{9} - 2 = 3$

c $\frac{x}{4} + 7 = 10$

d $\frac{x}{2} - 11 = 6$

e $\frac{x}{7} - 5 = 6$

f $\frac{x}{3} + 1 = 1$

g $\frac{x}{5} + 2.3 = 4.9$

h $\frac{x}{4} - \frac{3}{11} = \frac{4}{11}$

i $\frac{x}{2} + \frac{1}{9} = \frac{4}{9}$

5 Solve the following equations by backtracking.

a $\frac{x+4}{3} = 6$

b $\frac{x-8}{7} = 3$

c $\frac{x-8}{7} = 10$

d $\frac{x+11}{2} = 6$

e $\frac{x-5}{7} = 0$

f $\frac{x+100}{17} = 23$

g $\frac{x+2.21}{1.4} = 4.9$

h $\frac{x-1}{5} = \frac{4}{7}$

i $\frac{x+1}{4} = \frac{3}{8}$

6 Use backtracking to find the solution to the following equations.

a $3x - 7 = 23$

b $4(x + 7) = 40$

c $\frac{x+6}{9} = 6$

d $\frac{x}{5} - 2 = 8$

e $5(x - 3) = 15$

f $\frac{x+3}{8} = 3$

g $6(x - 4) = 18$

h $\frac{x}{3} + 10 = 12$

i $\frac{x}{2.1} - 1.7 = 3.6$

j $4x + \frac{3}{5} = 1$

k $\frac{x+5}{3} - 3 = 7$

l $3(2x + 5) = 21$

m $4(x - 2) + 5 = 21$

n $3\left(\frac{x}{2} + 1\right) = 15$

o $2(3x + 4) - 5 = 15$

7 Simplify and then solve the following equations by backtracking.

a $2x + 7 + 3x + 5 = 27$

b $x + x + 1 + x + 2 = 18$

c $3x + 9 + x - 4 = 17$

d $3x + 5x + 2x = 40$

e $6x + 6 - x - 4 = 37$

f $3x - 11 + 4x = 17$

g $5x - 2x + 5 - x = 19$

h $2x + 3x + 4x + 5 = 7$

i $7x - 4x + 8 - x = 10$



Solving equations



Solving equations

WORKED Example
7a



Solving equations

WORKED Example
7b



6.5



6.1

WORKED Example
8



What was the “Aviso Relation Oder Zeitung” by German, A. von Sohne in 1609?

Solve the equations below to discover the puzzle answer code.

$2A + 3 = 13$

$3(E - 20) = 12$

$(E - 25) \div 5 = 1$

$\frac{E+6}{3} = 5$

$\frac{7H}{2} = 7$

$3(K - 12) = 0$

$12L + 16 = 100$

$4 + 2N = 30$

$50 - 2N = 4$

$19 + 3P = 100$

$7R + 7 = 49$

$3S + 5 = 35$

$13T + 7 = 20$

$\frac{W}{5} + 4 = 7$

$\frac{W+5}{5} = 6$

$\frac{A-13}{5} = 3$

$\frac{E}{9} + 6 = 8$

$7E + 8 = 29$

$2E + 3E = 20$

$\frac{1}{2}(E + 1) = 10$

$\frac{I+12}{10} = 2$

$20 + 4K = 100$

$22 - L = 1$

$\frac{N}{8} + 3 = 5$

$\frac{O}{7} - 1 = 1$

$30 - P = 1$

$2R - 59 = 3$

$4(S - 20) = 24$

$\frac{T-8}{3} = 1$

$W + 2W = 51$

$\frac{Y}{11} - 2 = 13$

10 QUICK QUESTIONS 1

- 1 Write an equation to represent the following puzzle.

I am thinking of a number. When I add 11 to it the answer is 15.

- 2 Solve the equation $\frac{k}{8} = 4$ by inspection.

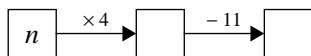
- 3 Complete the following flow chart to find the output number.



- 4 Use backtracking and inverse operations to find the input number in this flow chart.



- 5 Build up an expression by following the instructions on the flow chart.



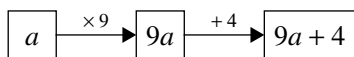
- 6 Build up an expression by following the instructions on the flow chart.



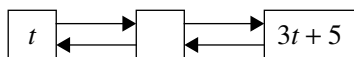
- 7 Draw the flow chart whose input number is m and whose output is given by the expression $3m + 2$.

- 8 Draw the flow chart whose input number is k and whose output is given by the expression $\frac{2k - 18}{6}$.

- 9 Complete this flow chart by writing in the operations that must be carried out in order to backtrack to a .



- 10 Complete this flow chart by writing in the operations that must be carried out in order to build the expression and backtrack to t .



Checking solutions

Hamish and Helen disagree about the solution to the equation $2x + 7 = 13$. Hamish says that the solution is $x = 10$, but Helen thinks that it is $x = 3$. They decide to check their solutions by looking at the value of $2x + 7$ and using substitution.

$$\begin{aligned} \text{If } x = 10, \quad \text{then } 2x + 7 &= 2 \times 10 + 7 \\ &= 27. \end{aligned}$$

$$\begin{aligned} \text{But if } x = 3, \quad \text{then } 2x + 7 &= 2 \times 3 + 7 \\ &= 13. \end{aligned}$$

The statement $2x + 7 = 13$ is true when $x = 3$. This means that $x = 3$ is the solution to the equation and so Helen is correct.

We can also test solutions of harder equations that we cannot solve by backtracking. Consider, for example, the equation $4x - 7 = 2x + 1$. Is the solution $x = 2$?

$$\begin{aligned} \text{If } x = 2, \text{ then } 4x - 7 &= 8 - 7 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{and } 2x + 1 &= 4 + 1 \\ &= 5. \end{aligned}$$

In this case $4x - 7$ does not equal $2x + 1$, so $x = 2$ is not the solution. Let us try $x = 3$.

$$\begin{aligned} \text{If } x = 3, \text{ then } 4x - 7 &= 12 - 7 \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{and } 2x + 1 &= 6 + 1 \\ &= 7. \end{aligned}$$

We see that $x = 3$ is not a solution either!

We could test many different values of x . A table is useful to summarise the results:

x	$4x - 7$	$2x + 1$
2	1	5
3	5	7
4	9	9
5	13	11

We can see from our table that $4x - 7 = 2x + 1$ when $x = 4$. So $x = 4$ is the solution to the equation $4x - 7 = 2x + 1$.

Guess, check and comment

We can now try to guess the solutions to some equations, even very difficult ones, because we are able to check our solutions. After each guess we must stop to think and comment so that our guesses improve and lead us towards the correct solution. A table is very useful to help us to organise our ideas.

Here is an example: Let us try to guess the solution to the equation $7x + 3 = 3x + 27$. We will need a table to write in our guess for x , the values of $7x + 3$ and $3x + 27$, and a comment. Don't be too concerned about making a good first guess because the most important thing is to start somewhere. Let us try $x = 1$.

If $x = 1$, then $7x + 3 = 10$ and $3x + 27 = 30$. Comparing the two values we note that the value obtained for $3x + 27$ is bigger. This is a useful comment.

Guess	Check		Comment
x	$7x + 3$	$3x + 27$	
1	10	30	$3x + 27$ is too big.

Try another guess such as $x = 4$.

If $x = 4$, then $7x + 3 = 31$ and $3x + 27 = 39$. We can see that the value obtained for $3x + 27$ is still too big, but the two values are closer together. We must be heading in the right direction!

Guess	Check		Comment
x	$7x + 3$	$3x + 27$	
1	10	30	$3x + 27$ is too big.
4	31	39	Getting closer

Next we will try $x = 8$.

If $x = 8$, then $7x + 3 = 59$ and $3x + 27 = 51$. The value obtained for $3x + 27$ is now too small. We have gone too far, but the number that we want is somewhere between 4 and 8.

Let us try $x = 6$.

If $x = 6$, then $7x + 3 = 45$ and $3x + 27 = 45$.

Guess	Check		Comment
x	$7x + 3$	$3x + 27$	
1	10	30	$3x + 27$ is too big.
4	31	39	Getting closer
8	59	51	$3x + 27$ is too small.
6	45	45	EUREKA!

So $x = 6$ is the solution to the equation $7x + 3 = 3x + 27$.

WORKED Example 9

For each of the following determine whether $x = 7$ is the solution to the equation.

a $\frac{x+5}{3} = 4$ **b** $2x - 8 = 10$

THINK

- a**
- 1 Write down the equation.
 - 2 Write down the left-hand side (LHS) of the equation and substitute $x = 7$.
 - 3 Compare with the right-hand side (RHS) of the equation.
 - 4 Comment on the result.
- b**
- 1 Write down the equation.
 - 2 Write down the left-hand side of the equation and substitute $x = 7$.
 - 3 Compare with the right-hand side of the equation.
 - 4 Comment on the result.

WRITE

a $\frac{x+5}{3} = 4$

$$\begin{aligned} \text{If } x = 7, \text{ LHS} &= \frac{x+5}{3} \\ &= \frac{7+5}{3} \\ &= 4 \end{aligned}$$

RHS = 4

The solution is $x = 7$.

b $2x - 8 = 10$

$$\begin{aligned} \text{If } x = 7, \text{ LHS} &= 2x - 8 \\ &= 2(7) - 8 \\ &= 14 - 8 \\ &= 6 \end{aligned}$$

RHS = 10.

Therefore, $x = 7$ is not the solution.

WORKED Example 10

For each equation below there is a solution given. Is the solution correct?

a $\frac{x+2}{3} = 2x - 12, x = 10$ **b** $3x - 7 = 2x + 3, x = 10$

THINK

- a**
- 1 Write down the equation.
 - 2 Write down the left-hand side of the equation and substitute $x = 10$.
 - 3 Write down the right-hand side of the equation and substitute $x = 10$.
 - 4 Comment on the results.


WRITE

a $\frac{x+2}{3} = 2x - 12$

$$\begin{aligned} \text{If } x = 10, \text{ LHS} &= \frac{x+2}{3} \\ &= \frac{10+2}{3} \\ &= \frac{12}{3} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 2x - 12 \\ &= 2(10) - 12 \\ &= 20 - 12 \\ &= 8 \end{aligned}$$

Therefore $x = 10$ is not the solution.

Continued over page 

THINK

- b**
- 1 Write down the equation.
 - 2 Write down the left-hand side of the equation and substitute $x = 10$.
 - 3 Write down the right-hand side of the equation and substitute $x = 10$.
 - 4 Comment on the results.

WRITE

b $3x - 7 = 2x + 3$
 If $x = 10$, LHS = $3x - 7$
 $= 3(10) - 7$
 $= 30 - 7$
 $= 23$

RHS = $2x + 3$
 $= 2(10) + 3$
 $= 20 + 3$
 $= 23$

The solution is $x = 10$.

WORKED Example 11

Use guess, check and comment to solve the equation $2x + 21 = 4x - 1$.

THINK

- 1 Set up a table and try substituting some values for x .

 Guess $x = 1$ and substitute.
 Guess $x = 6$ and substitute.
 Guess $x = 10$ and substitute.
 Only a small change is needed.
- 2 State the solution.

WRITE

Guess	Check		Comment
x	$2x + 21$	$4x - 1$	
1	23	3	$4x - 1$ is too small.
6	33	23	This is closer.
10	41	39	Very close
11	43	43	That's it!

The solution is $x = 11$.

WORKED Example 12

Find two numbers whose sum is 31 and whose product is 150.

THINK

- 1 The numbers add up to 31 so guess two numbers that do this. Then check by finding their product.

 Guess 1 and 30.
 Guess 10 and 21.
 Try a number between 1 and 10 for the first number.
 Try a number between 5 and 10 for the first number.
 Try a number between 5 and 8 for the first number.
- 2 State the solution.

WRITE

Guess	Check	Comment
Sum (small number first)	Product (P)	
1, 30	$1 \times 30 = 30$	P is too low.
10, 21	$10 \times 21 = 210$	P is too high.
5, 26	$5 \times 26 = 130$	P is too low.
8, 23	$8 \times 23 = 184$	P is too high.
6, 25	$6 \times 25 = 150$	That's it!

The numbers are 6 and 25.

remember

We can check our solution to an equation by substituting the value for the pronumeral to see if it is true.

EXERCISE 6D

Checking solutions



WORKED Example

9

1 For each of the following determine whether:

- a $x = 3$ is the solution to the equation $x + 2 = 6$
- b $x = 3$ is the solution to the equation $2x - 1 = 5$
- c $x = 5$ is the solution to the equation $2x + 3 = 7$
- d $x = 4$ is the solution to the equation $6x - 6 = 24$
- e $x = 10$ is the solution to the equation $3x + 5 = 20$
- f $x = 5$ is the solution to the equation $4(x - 3) = 8$
- g $x = 7$ is the solution to the equation $3(x - 2) = 25$
- h $x = 8$ is the solution to the equation $5(x + 1) = 90$
- i $x = 12$ is the solution to the equation $6(x - 5) = 42$
- j $x = 81$ is the solution to the equation $3x - 53 = 80$
- k $x = 2.36$ is the solution to the equation $5x - 7 = 4.8$
- l $x = 4.4$ is the solution to the equation $7x - 2.15 = 18.64$.

2 Complete the table below to find the value of $2x + 3$ when $x = 0, 1, 2, 3, 4$.

x	0	1	2	3	4
$2x + 3$					

- a For what value of x does $2x + 3 = 11$?
- b What is the solution for $2x + 3 = 11$?
- c What is the solution for $2x + 3 = 5$?

3 Complete the table below to find the value of $5(x - 2)$ when $x = 2, 3, 4, 5, 6$.

x	2	3	4	5	6
$5(x - 2)$					

- a What is the solution to $5(x - 2) = 10$?
- b What is the solution to $5(x - 2) = 20$?
- c What do you guess is the solution to $5(x - 2) = 30$? Check your guess.



4 a Copy and complete this table.

b What is the solution to $2x + 1 = 3x - 5$?

x	$2x + 1$	$3x - 5$
3	7	
4		
5		10
6		
7		



5 a Copy and complete this table.

b What is the solution to

$$\frac{x+3}{2} = 2x - 6?$$

x	$\frac{x+3}{2}$	$2x - 6$
3	3	
5		
7		8
9		
11		



6 For each equation below there is a solution given. Is the solution correct?

- | | | |
|---|---------------------------------|------------|
| a | $2x + 1 = 3x - 5$ | $x = 6$ |
| b | $5x + 1 = 2x - 7$ | $x = 8$ |
| c | $3x - 5 = x + 8$ | $x = 10$ |
| d | $5x = 2x + 12$ | $x = 4$ |
| e | $4x = 3x + 8$ | $x = 8$ |
| f | $3x = x + 20$ | $x = 15$ |
| g | $3x - 1.2 = x + 2.9$ | $x = 1.9$ |
| h | $6x + 1.5 = 2x + 41.5$ | $x = 10$ |
| i | $2.4(x + 1) = 9.6$ | $x = 3$ |
| j | $1.2(x + 1.65) = 0.2(x + 9.85)$ | $x = 0.45$ |



7 Use guess, check and comment to solve the equation $3x + 11 = 5x - 1$. The process has been started for you.

Guess	Check		Comment
x	$3x + 11$	$5x - 1$	
1	14	4	$5x - 1$ is too small.
10	41	49	$5x - 1$ is too big.
5			



8 Use guess, check, comment to solve the following equations.

- | | | | | | |
|---|--------------------|---|-----------------------|---|---------------------|
| a | $5x + 15 = x + 27$ | b | $2x + 12 = 3x - 2$ | c | $x + 20 = 3x$ |
| d | $12x - 18 = 10x$ | e | $10(x + 1) = 5x + 25$ | f | $x(x + 1) = 21x$ |
| g | $x(x + 7) = 12x$ | h | $6(x - 2) = 4x$ | i | $3(x + 4) = 5x + 4$ |



**WORKED
Example**
12

- 9 Find two numbers whose sum is 21 and whose product is 98. The process has been started for you.

Guess	Check	Comment
Sum (small number first)	Product (P)	
1, 20	$1 \times 20 = 20$	P is too low.
10, 11	$10 \times 11 = 110$	P is too high.

- 10 Use guess, check, comment to find two numbers whose sum (S) and product (P) are given:
- | | | | |
|---|------------------------------|---|-----------------------------|
| a | sum = 26, product = 165 | b | sum = 27, product = 162 |
| c | sum = 54, product = 329 | d | sum = 45, product = 296 |
| e | sum = 178, product = 5712 | f | sum = 104, product = 2703 |
| g | sum = 153, product = 4662 | h | sum = 242, product = 14 065 |
| i | sum = 6.1, product = 8.58 | j | sum = 8, product = 14.79 |
| k | sum = 978, product = 218 957 | l | sum = 35, product = 274.89. |
- 11 Copy and complete this table by substituting each x value into $x^2 + 4$ and $4x + 1$. The first row has been completed for you. Use the table to find a solution to the equation $x^2 + 4 = 4x + 1$. (Remember, 5^2 means $5 \times 5 = 25$.)

x	$x^2 + 4$	$4x + 1$
0	4	1
1		
2		9
3	13	
4		



- 1 A football team won 4 more games than it lost. The team played 16 games. How many did it win?
- 2 Lily is half the age of Pedro. Ross is 6 years older than Lily and 6 years younger than Pedro. How old is Pedro?
- 3 A plumber cut a 20 metre pipe into 2 pieces. One of the pieces is 3 times longer than the other. What are the lengths of the two pieces of pipe?
- 4 Julie has the same number of sisters as brothers. Her brother Todd has twice as many sisters as brothers. How many children are in the family?

10 QUICK QUESTIONS 2

- Write the inverse operation for $+ 8$.
- Draw a flow chart to represent this puzzle and then solve by backtracking.
I am thinking of a number. When I multiply it by 2 and then add 9 the answer is 43.
- Draw a flow chart and use backtracking to find the solution to the equation $12k - 8 = 124$.
- Solve the equation $4(a - 7) = 48$ by backtracking.
- Solve the equation $\frac{e}{8} + 2 = 10$ by backtracking.
- First simplify, then solve the equation $2m + 9 + 4m - 4 = 41$.
- Determine whether this statement is true or false: $w = 30$ is the solution to $\frac{w}{10} + 5 = 8$.
- Complete the table below to find the value of $7(2a + 5)$ when $a = 0, 1, 2, 3, 4, 5$.

a	0	1	2	3	4	5
$7(2a + 5)$						

What is the solution to $7(2a + 5) = 63$?

- Complete this table.

a	$3a + 4$	$5a - 10$
5		
6		
7		
8		
9		

What is the solution to $3a + 4 = 5a - 10$?

- Use guess, check and comment to solve the equation $9m + 8 = 11m - 30$.

Guess	Check		Comment
m	$9m + 8$	$11m - 30$	

Solving word problems

Equations can be used to solve many everyday puzzles and problems. The trick is to change word problems into equations. We have done this with ‘I am thinking of a number’ problems, but many other situations can be described by equations as well. Here is an example.

Luke and Stefan play cricket in the same team. Last week they scored a total of 53 runs, but Luke scored 11 more runs than Stefan. How many runs did Luke score?

Step 1. Read the question carefully.

The important points are these:

Luke scored 11 more runs than Stefan.

Luke’s score plus Stefan’s score equals 53.

Step 2. Turn the information into an equation.

We are asked to find Luke’s score, so we will call it x . We usually say ‘Let x equal Luke’s score’. Stefan scored 11 runs fewer than Luke so his score will be $x - 11$. Since their scores total 53 we know that $x + x - 11 = 53$.

Step 3. Simplify and then solve the equation $x + x - 11 = 53$. This is the same as $2x - 11 = 53$.

We can backtrack or solve by inspection to find $x = 32$.

Step 4. Check your solution. If Luke scored 32 runs, then Stefan scored 21 runs and their total is $32 + 21 = 53$. This fits the question.

Step 5. Write your answer in words. Luke scored 32 runs.

WORKED Example 13

If x is a whole number, write down the value of:

- a** the number which is 10 more than x
- b** the number which is one-third of x
- c** the number of minutes in x hours.

THINK

- a** To get 10 more means to add 10.
- b** To find one-third means to divide by 3.
- c** To change hours to minutes, multiply by 60.

WRITE

- a** $x + 10$
- b** $x \div 3 = \frac{x}{3}$
- c** $60x$

WORKED Example 14

Simone earns \$5 per week more than Jessica. If Jessica earns \$ x , find their total earnings per week.

THINK

- 1** Jessica earns x . Simone earns more, so she earns $x + 5$.
- 2** Simplify.
- 3** State the answer.

WRITE

$x + x + 5$
 $= 2x + 5$
 Their total earnings are $\$2x + 5$.

WORKED Example 15

Anh is 2 years older than his sister Phuong and the sum of their ages is 30 years. How old is Phuong?

THINK

- 1 Identify what is needed and represent this with x . We need to find Phuong's age.
- 2 Form an expression using the information that Anh is 2 years older.
- 3 Form an equation showing that their ages add up to 30.
- 4 Simplify the equation.
- 5 Backtrack to solve for x .

- 6 Check the solution.
- 7 Write the answer in words.

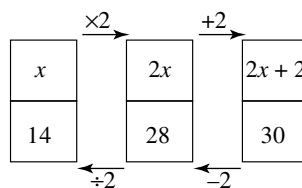
WRITE

Let x equal Phuong's age.

Anh's age is $x + 2$.

$$x + x + 2 = 30$$

$$2x + 2 = 30$$



$$x = 14$$

Check: Anh is 16 and $16 + 14 = 30$.

Phuong is 14 years old.

remember

Solving word problems involves 5 steps.

1. Read the question carefully.
2. Turn the information into an equation.
3. Simplify and then solve the equation.
4. Write your answer in words.
5. Check your solution.

EXERCISE 6E**Solving word problems****WORKED Example 13**

1 If x is a whole number, write down the value of:

- a the number which is 5 more than x
- b the number which is 8 less than x
- c the number which is twice as large as x
- d the number which is one quarter of x
- e the next largest number after x
- f the number of cents in x dollars
- g the number of metres in x centimetres
- h the number of days in x weeks
- i the (mean) average of x and 20
- j how much it costs to rent a tennis court for x hours at \$10 per hour
- k the average of 5, 18 and k .



2 **multiple choice**

- a John is x years old. In 5 years his age will be:
A $5x$ **B** 17 **C** $x - 5$ **D** $x + 5$ **E** 10
- b Chris is 2 years older than Jenny. If Jenny is x years old then Chris's age is:
A $x - 2$ **B** $2x$ **C** $x + 2$ **D** 2 **E** $\frac{x}{2}$
- c Duy earns \$5 per week more than Ben. If Duy earns \$ x , then Ben earns:
A $x + 5$ **B** $5x$ **C** 10 **D** $x - 5$ **E** $5 - x$
- d Assume x is an even number. The next largest even number is:
A $x + 1$ **B** $x + 2$ **C** 4 **D** $x + 4$ **E** $2x$
- e Assume x is an odd number. The next largest odd number is:
A $x + 1$ **B** $x + 2$ **C** 3 **D** $x + 3$ **E** $2x + 1$
- f Gian is half Carla's age. If Gian is x years old, then Carla's age is:
A $\frac{x}{2}$ **B** $x - \frac{1}{2}$ **C** $x + \frac{1}{2}$ **D** $2x$ **E** $\frac{1}{2}x$

**WORKED
Example**

14

- 3 a Ali earns \$7 more per week than Halit. If Halit earns \$ x , find their total earnings per week.
 b Sasha is twice as old as Kapila. If Kapila is x years old, find their total age.
 c Frank has had 3 more birthdays than James. If James is $2x$ years old, find their total age.
 d If x is the smaller of 2 consecutive whole numbers, find the sum of the 2 numbers.
 e If x is the smallest of 3 consecutive odd numbers, find the sum of the 3 numbers.
 f If x is the smallest of 3 consecutive even numbers, find the sum of the 3 numbers.

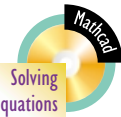


Use equations to solve the following questions.

**WORKED
Example**

15

- 4 Imraan is 5 years older than his brother Gareth and the sum of their ages is 31 years. How old is Gareth? (Let x represent Gareth's age.)
- 5 In 3 basketball games Karina has averaged 12 points each game. In the first game she scored 11 points, in the second she scored 17 points, and in the third game she scored x points.
 a From the given information, what is the average of 11, 17 and x ?
 b Write an equation using the answer to part a.
 c Solve the equation.
 d How many points did Karina score in the third game?
- 6 Melanie and Callie went tenpin bowling together. Melanie scored 15 more pins than Callie, and their total score was 207. What did Callie score?
- 7 The sum of 3 consecutive whole numbers is 51. Find the numbers. (*Hint*: Let the smallest number equal x .)
- 8 The sum of 3 consecutive odd numbers is 27. What are the numbers?
- 9 Three consecutive multiples of 5 add up to 90. What are the 3 numbers?
- 10 David is 5 years younger than his twin brothers. If the sum of their ages is 52, then how old is David?
- 11 In the high jump event Chris leapt 12 centimetres higher than Tim, but their 2 jumps made a total of 3 metres. How high did Chris jump?



12 Daniel and Travis are twins, but their sister Phillipa is 3 years older. If the sum of their three ages is 36, how old are the twins?

- 13 a Write a rule to describe how you would calculate the total cost of these 3 items (that is, the hamburger and the two glasses of soft drink).
- b Let d represent the cost of a drink. If the hamburger costs \$3.75 and the total cost is \$7.25, write an equation which links all this information.
- c Solve the equation to find the cost of a drink.



14



WINDSURFER HIRE
\$50 deposit
\$12 per hour

- a How much will it cost to hire a windsurfer for:
- i 1 hour? ii 2 hours? iii 3 hours?
- b Write a rule that could be used to calculate the cost of hiring a windsurfer for n hours.
- c Use the rule to calculate the cost of hiring a windsurfer for 8 hours.
- d You have \$100 to spend. Write an equation to help you work out how many hours you could hire a windsurfer for.
- e Solve the equation to find the number of hours for which you could hire the windsurfer.
- f Work out how much money (if any) you would have left over when you pay the hire charge.
- g After a great day's windsurfing, you have returned the windsurfer (with no damage). Would you have enough money to buy fish and chips on the way home?

Kids' hotline walk-a-thon



Your friend Alexandra has raised \$75.00 for the 'Kids' hotline'. Can you work out how far she has walked? Look at her sponsor form above.

- Let k represent the number of kilometres Alexandra has walked. Write an expression for the amount of money James McLennan would need to pay her.
- In a similar way, write an expression for the amount of money each person listed on her sponsor form would need to pay.
- Write a rule showing the total amount of money raised by Alexandra if she walked k kilometres in the walk-a-thon.
- If the total amount of money raised by Alexandra is \$75.00, write an equation which could be solved to find the value of k .
- Solve the equation to find k and write a sentence to state your answer.
- If instead of raising \$75.00, Alexandra has raised \$120.00 from her sponsors, how far would she have walked?

Another friend of yours had the following sponsor form.

Sponsor's name	Address	City	Amount per km
1 Megan Talty	26 Toy Rd	Rosanna	\$1.50
2 Michael Lamb	121 Holston St	Hurstbridge	\$3.50
3 Jessica Bacash	74 Mason Dr	Preston	\$2.00
4 Dinesh Jones	5 Eagle St	Hawthorn	\$2.50

- Daniel is not sure whether he should collect \$82.50 or \$85.50 from his sponsors. Using equations, work out which amount he should collect.
- Find the total number of kilometres Daniel walked in the walk-a-thon.
- If Daniel wanted to raise over \$100 for the 'Kids' hotline' how far would he have needed to walk? (Give the answer to the nearest kilometre.)

summary

Copy the sentences below. Fill in the gaps by choosing the correct word or expression from the word list that follows.

- 1 _____ help us to organise harder equations.
- 2 We can use _____ and inverse operations to solve equations.
- 3 We can use flow charts to build up _____ expressions.
- 4 As well as building up an algebraic expression using a flow chart, we can backtrack to our input number using _____ operations.
- 5 The inverse operation to multiplying is _____.
- 6 The inverse operation to dividing is _____.
- 7 The inverse operation to adding is _____.
- 8 The inverse operation to subtracting is _____.
- 9 We can _____ equations by drawing a flow chart and backtracking.
- 10 We can check our solution to an equation by _____ the value for the pronumeral to see if it is true.
- 11 Changing _____ problems into equations involves the 5 steps shown below.
 - Step 1 _____ the question carefully.
 - Step 2 Turn the information into an _____.
 - Step 3 _____ and then solve the equation.
 - Step 4 _____ your solution.
 - Step 5 Write your _____ in words.

WORD LIST

substituting
inverse
dividing
equation

flow charts
multiplying
check
solve

subtracting
backtracking
simplify
adding

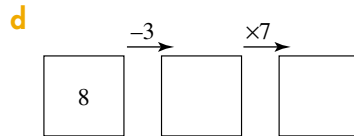
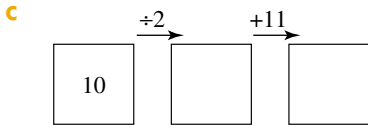
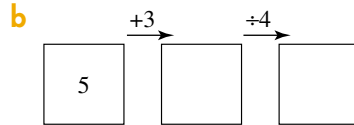
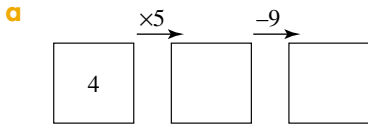
answer
read
algebraic
word

CHAPTER review

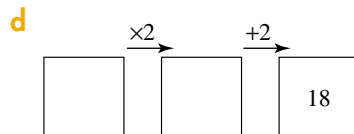
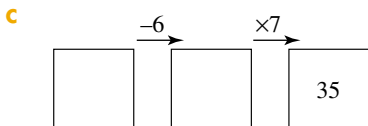
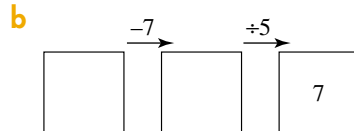
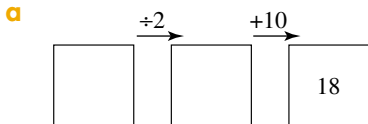
1 Solve these equations by inspection.

a $m + 7 = 12$ b $5h = 30$ c $s - 12 = 7$ d $\frac{d}{5} = 4$

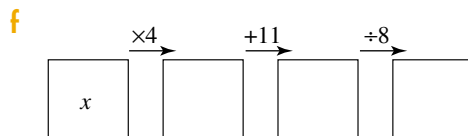
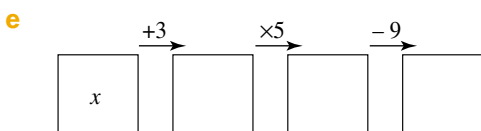
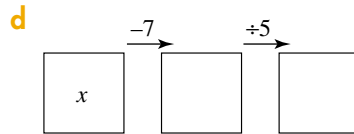
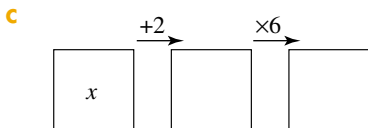
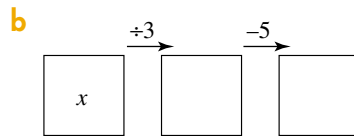
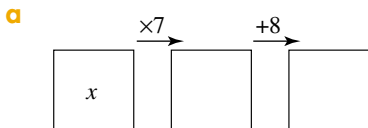
2 Complete these flow charts to find the output number.



3 Use backtracking and inverse operations to find the input number in each of these flow charts.



4 Build up an expression by following the instructions on the flow chart.



5 Draw the flow chart whose input is x and whose output is given by the expression.

a $5(x + 7)$ b $\frac{x}{4} - 3$ c $6x - 14$ d $\frac{x + 2}{5}$

6A

6A

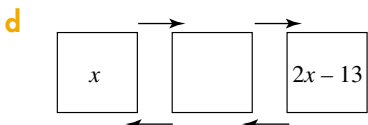
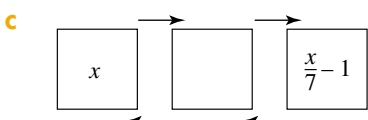
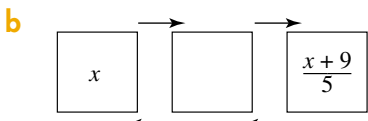
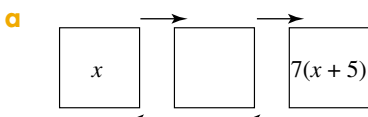
6A

6B

6B

6B

- 6 Complete these flow charts by writing in the operations which must be carried out in order to backtrack to x .



6C

- 7 Draw a flow chart and use backtracking to find the solution to the following equations.

a $7x + 6 = 20$ b $9(y - 8) = 18$ c $\frac{m}{5} - 3 = 9$ d $\frac{s+7}{5} = 5$

6C

- 8 Use backtracking to find the solution to these equations.

a $3(d + 1) = 15$ b $\frac{t}{4} - 11 = 14$ c $6d - 3 = 15$ d $\frac{a+6}{4} = 3$

6C

- 9 Simplify the expression and then solve the equation for each of the following.

a $7v + 3 + 3v + 4 = 37$ b $6c + 15 - 5c - 8 = 19$

6D

- 10 For each equation below there is a solution given. Is the solution correct?

a $5x - 7 = 2x + 2$ $x = 3$ b $\frac{x+9}{2} = 2x - 7$ $x = 5$

6D

- 11 Use guess, check and comment to find two numbers whose sum and product are given.

a Sum = 83, product = 1632 b Sum = 86, product = 1593

6E

- 12 Sophie and Jackie each have a collection of football cards. Jackie has 5 more cards than Sophie, and together they have 67 cards. By writing and solving an equation, find out how many cards Sophie owns.

6E

- 13 Andreas has completed 2 more pieces of homework than Richard who submitted x pieces of homework for the semester. If the total number of pieces of homework submitted by the two boys is 12, how many pieces of homework did Andreas submit?

